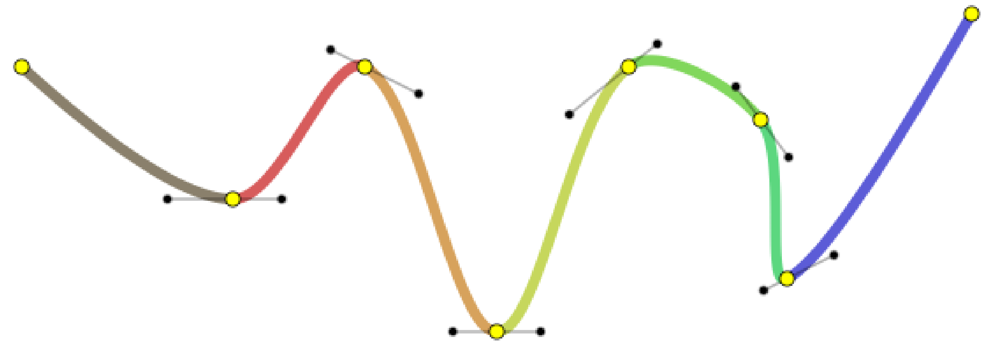
Spline

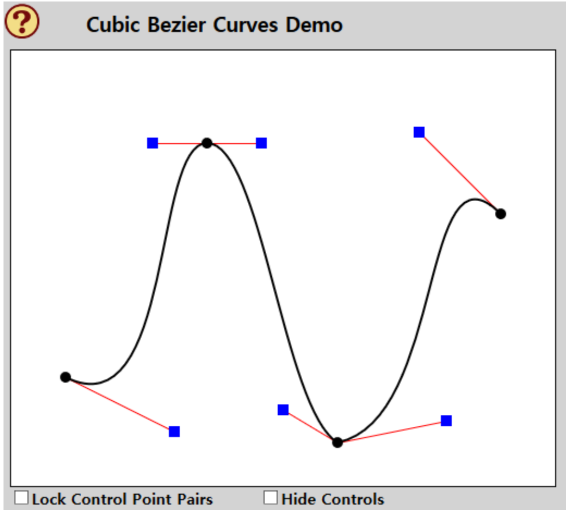
- Spline: ***piecewise* polynomial**



- Three issues:
  - How to connect these pieces *continuously*?
  - How easy is it to *"control"* the shape of a spline?
  - Does a spline have to *pass through* specific points?

Continuity

- Let’s try another Bezier demo: Bezier spline



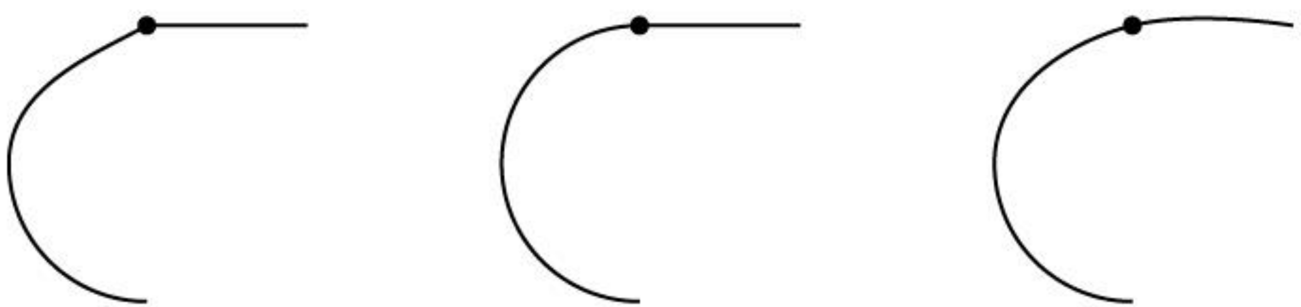
http://math.hws.edu/graphicsbook/demos/c2/cubic-bezier.html

- How to “smooth” the spline?

Continuity

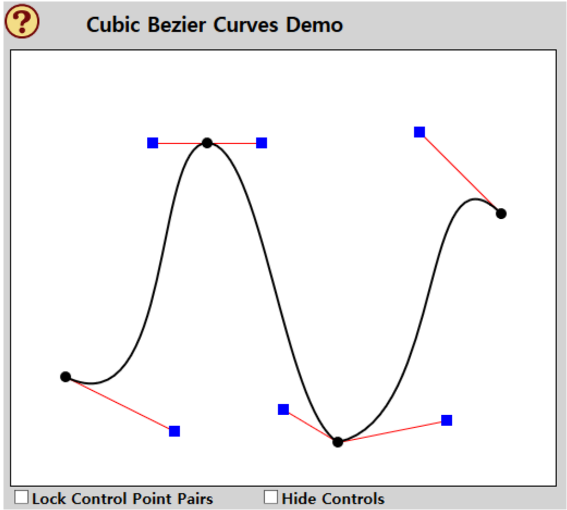
- Smoothness can be described by degree of continuity
  - zero-order (C^0): position matches from both sides
  - first-order (C^1): position and 1st derivative (velocity) match from both sides
  - second-order (C^2): position and 1st & 2nd derivatives (velocity & acceleration) match from both sides

zero order   first order   second order

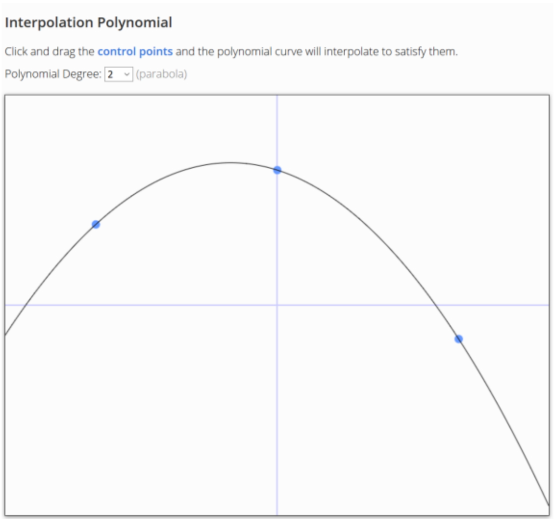


Control

- Let's say you want to make a specific shape using these two curves. Which one is more controllable?



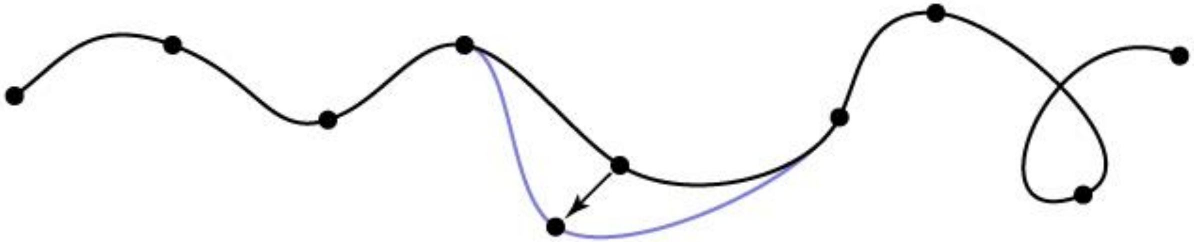
http://math.hws.edu/graphicsbook/demos/c2/cubic-bezier.html



https://www.benjoffe.com/code/demos/interpolate

Control

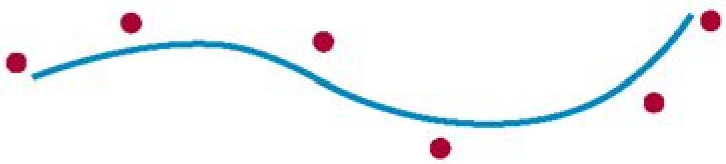
- Local control
  - changing control point only affects a **limited part** of spline
  - without this, splines are very difficult to use
  - many likely formulations lack this
    - natural spline
    - polynomial fits



Interpolation / Approximation

- Interpolation: passes through points

- Approximation: merely guided by points



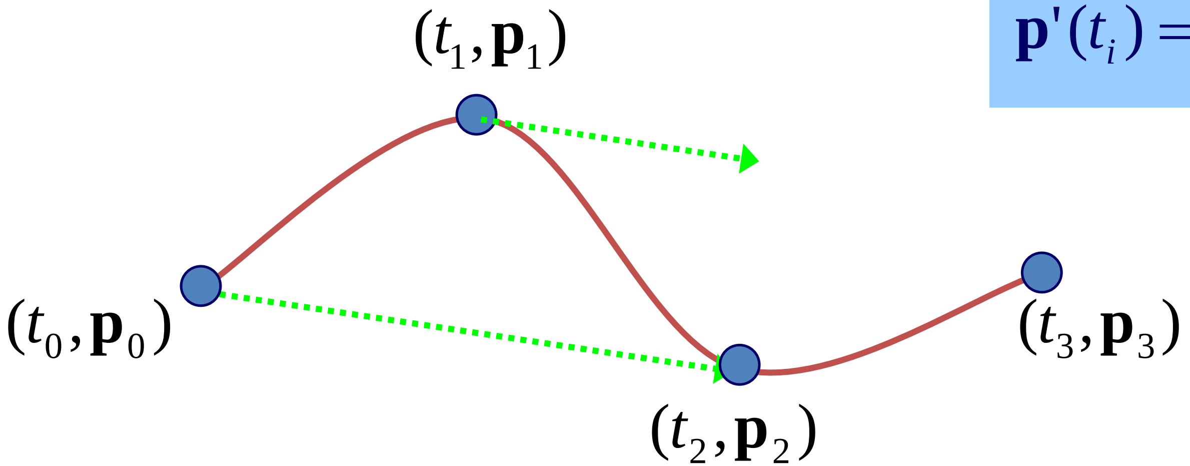
- Interpolation properties are preferable, but not mandatory

Catmull-Rom Spline

- A Bezier or Hermite curve interpolates two end points only
- What if we want a cubic spline interpolating all control points?

- Catmull-Rom Splines
  - One Hermite curve between two consecutive control points
  - Define end point derivatives using adjacent control points

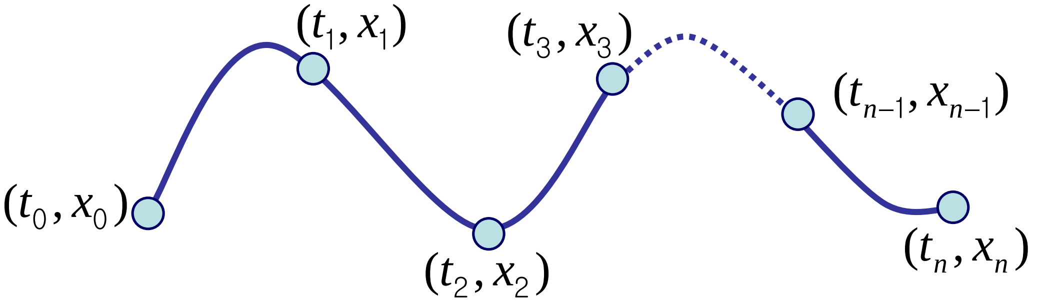
$$\mathbf{p}'(t_i) = \frac{\mathbf{p}_{i+1} - \mathbf{p}_{i-1}}{2}$$



- C^1 continuity, local controllability, interpolation

Natural Cubic Splines

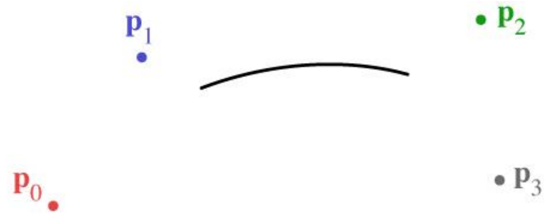
- We want to achieve higher continuity (at least C^2)
- *4n* unknowns
  - *n* Bezier curve segments (4 control points per each segment)
- *4n* equations
  - *2n* equations for end point interpolation
  - (*n-1*) equations for tangential continuity
  - (*n-1*) equations for second derivative continuity
  - 2 equations: $x''(t_0) = x''(t_n) = 0$



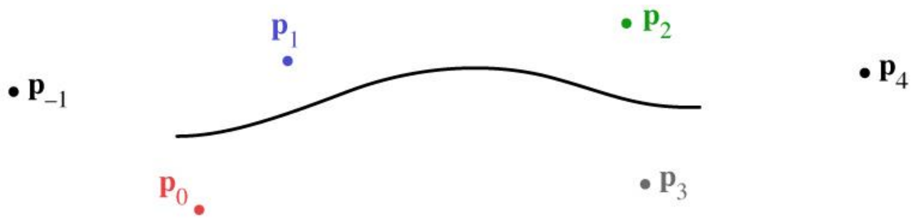
- C^2 continuity, no local controllability, interpolation

B-splines (brief intro)

- Use 4 points, but approximate only middle two



- Draw curve with overlapping segments
  - 0-1-2-3, 1-2-3-4, 2-3-4-5, 3-4-5-6, etc



- C^2 continuity, local controllability, approximation